Assignment - 3

Q.1 Explain the partial Differentiation with example.

Let $z = f(x,y)$ be function of two individual variables x and y the derivative with respect to x keeping y constant is called partial derivative of z with respect to x

It is denoted by $\frac{\partial z}{\partial x}$, $\frac{\partial f}{\partial x}$, f_x ......

It is defined as

$$\partial z/\partial x = \lim_{\partial x \to 0} f(x+\partial x, y) - f(x,y) / \partial x$$

Q.2 If $u = \log(x^3+y^3+z^3-3xyz)$ Then prove that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = \frac{3}{x+y+z}$

$$u = \log(x^3+y^3+z^3-3xyz)$$

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} - \frac{3}{x+y+z}$$

$$e^u = x^3+y^3+z^3-3xyz$$

P.d. w.r. to x

$$e^u . u_x = 3x^2 - 3yz$$

Smilarly

$$e^u . u_y = 3y^2 - 3xz$$

$$e^u . u_z = 3z^2 - 3xy$$

$$e^u \left[\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z}\right] = 3 \cdot \left[x^2+y^2+z^2-xy-yz-zx\right]$$

$$\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}=\frac{3(x^2+y^2+z^2-xy-yz-zx)}{(x^3+y^3+z^3-3xyz)}$$

$$\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}=\frac{3\cancel{(x^2+y^2+z^2-xy-yz-zx)}}{(x+y+z)\cancel{(x^2+y^2+z^2-xy-yz-zx)}}$$

$$\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}=\frac{3}{x+y+z}$$ proved

Q.3 If $u=\sin^{-1}\frac{x^2+y^2}{x+y}$; Then prove that

$$x\frac{\partial f}{\partial x}+y\frac{\partial u}{\partial y}=\tan u$$

$u=\sin^{-1}\frac{x^2+y^2}{x+y}$ is a homogeneous function of x and y of degree 1.

$$x\frac{\partial f}{\partial x}+y\frac{\partial u}{\partial y}=\tan u$$

Ten $z=\sin u$

Now applying Euler's theorem on z, we get

$$x\frac{\partial z}{\partial x}+y\frac{\partial z}{\partial y}=z \quad \text{---} (1)$$

putting the value of z in equation (i) we get

$$x\frac{\partial \sin u}{\partial x}+y\frac{\partial \sin u}{\partial y}=\sin u$$

$$x\cos u\frac{\partial u}{\partial x}+y\cos u\frac{\partial u}{\partial y}=\sin u$$

$$\frac{x\partial u}{\partial x} + \frac{y\partial u}{\partial y} = \frac{\sin u}{\cos u}$$

$$x\frac{\partial u}{\partial x} + \frac{y\partial u}{\partial y} = \tan u$$

proved

Q.4 If f is a homogeneous function of degree n then

$$y\frac{\partial^2 f}{\partial y^2} + \frac{x\partial^2 f}{\partial x\partial y} = (n-1)\frac{\partial f}{\partial y}$$

Sol F is a homogeneous function of degree n.

By Eular's Theorem

$$\frac{x\partial f}{\partial x} + \frac{y\partial f}{\partial y} = nzf$$

Now, partial deriative of this It w.r. of x, y.

$$\frac{x\partial^2 f}{\partial x\partial y} + y\frac{\partial^2 f}{\partial y^2} + \frac{\partial f}{\partial y}(1) = \frac{n\partial f}{\partial y}$$

$$x\frac{\partial^2 f}{\partial x\partial y} + \frac{y\partial^2 f}{\partial y^2} = \frac{n\partial f}{\partial y} - \frac{\partial f}{\partial y}$$

$$\frac{y\partial^2 f}{\partial y^2} + \frac{x\partial^2 f}{\partial x\partial y} = (n-1)\frac{\partial f}{\partial y}$$ proved

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Q.5 If f is a homogeneous function of degree n then

$$x^2\frac{\partial^2 f}{\partial x^2}+y^2\frac{\partial^2 f}{\partial y^2}+2xy\frac{\partial^2 f}{\partial x\partial y}=n(n-1)f$$

Sol Given that
f is a homogeneous function of degree n

By Eular's theorem

$$x\frac{\partial f}{\partial x}+y\frac{\partial f}{\partial y}=nf \quad \text{--- (1)}$$

Differentiate w.r. of x

$$x\frac{\partial^2 f}{\partial x^2}+\frac{\partial f}{\partial x}\cdot(1)+y\frac{\partial f}{\partial x\partial y}=\frac{n\partial f}{\partial x}$$

Multiple by x

$$\frac{x^2\partial^2 f}{\partial x^2}+\frac{x\partial f}{\partial x}+\frac{xy\partial^2 f}{\partial x\partial y}=nx\frac{\partial f}{\partial x}$$

$$\frac{x^2\partial^2 f}{\partial x^2}+\frac{xy\partial^2 f}{\partial x\partial y}=nx\frac{\partial f}{\partial x}-\frac{x\partial f}{\partial x}$$

$$\frac{x^2\partial^2 f}{\partial x^2}+xy\frac{\partial^2 f}{\partial x\partial y}=(n-1)\frac{x\partial f}{\partial x} \quad \text{--- (2)}$$

Similarly for y

$$y^2\frac{\partial^2 f}{\partial y^2} + xy\frac{\partial^2 f}{\partial x \partial y} = (n-1)\, y\frac{\partial f}{\partial y} \quad (3)$$

Now equ (1) + equ (2)

$$x^2\frac{\partial^2 f}{\partial x^2} + y^2\frac{\partial^2 f}{\partial y^2} + 2xy\frac{\partial^2 f}{\partial x \partial y} = (n-1)\left[x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y}\right]$$

$$x^2\frac{\partial^2 f}{\partial x^2} + y^2\frac{\partial^2 f}{\partial y^2} + 2xy\frac{\partial^2 f}{\partial x \partial y} = n(\overset{n}{\cancel{x}}-1)\, f$$

Proved